# Pairs Trading Project:
# An application on US and Swiss Equity Markets

Moutie Mohamed Ali, Guillen Steulet, Jules Broglin

## 1 Introduction

Pairs trading is a market-neutral statistical arbitrage strategy that relies on the mean-reverting behavior of a linear combination of two related financial assets. The aim of this project is twofold:

- Extend the vanilla Pairs Trading strategy to more advanced and calibrated methods
- Apply it to US and Swiss equity markets

## 2 Vanilla Pairs Trading

### 2.1 Theoretical Foundation

The core idea behind pairs trading is that two related assets may share a stable long-run equilibrium relationship. While each price series may be non-stationary on its own, a linear combination of the two may be stationary. This is the notion of cointegration, and it provides the theoretical basis for mean-reversion trading.

Given two asset prices $P_{A,t}$ and $P_{B,t}$, we define the spread as

$$S_t = P_{A,t} - \beta P_{B,t},$$

where $\beta$ is the hedge ratio. Intuitively, $\beta$ determines how much of asset $B$ is needed to hedge one unit of asset $A$ and construct a market-neutral relative-value position.

In the vanilla implementation, we begin with a static hedge ratio estimated using OLS, then monitor the spread through a rolling normalization:

$$Z_t = \frac{S_t - \mu_t}{\sigma_t},$$

where $\mu_t$ and $\sigma_t$ are the rolling mean and standard deviation of the spread over a fixed lookback window.

The trading logic is simple:

- **Long spread:** if $Z_t < -z^\star$, the spread is abnormally low, so we buy asset $A$ and short asset $B$;
- **Short spread:** if $Z_t > z^\star$, the spread is abnormally high, so we short asset $A$ and buy asset $B$;
- **Exit:** positions are closed when the spread reverts toward its mean.

This provides a clean baseline strategy, but it also has important limitations. In particular, the hedge ratio is assumed constant, spread dynamics are summarized only by a rolling $z$-score, and structural breaks or regime changes may invalidate the mean-reversion assumption.

## 3 Extensions

### 3.1 Extending the hedge ratio with Kalman Filters

A first extension is to allow the hedge ratio to vary over time rather than remain fixed over the whole sample. In practice, the relationship between two assets changes with market regimes, liquidity, volatility, and fundamentals. A static OLS estimate may therefore become quickly outdated.

To address this, we plan to model the price relation in a state-space framework:

$$P_{A,t} = \alpha_t + \beta_t P_{B,t} + \varepsilon_t,$$

where $(\alpha_t, \beta_t)$ are latent time-varying states estimated sequentially using a Kalman filter.

This approach should provide a more adaptive spread construction and potentially improve signal stability. One of our key questions is whether a dynamic hedge ratio leads to better out-of-sample performance than a static one once transaction costs and turnover are taken into account.

### 3.2 Ornstein-Uhlenbeck process as a model for the spread

A second extension is to explicitly model the spread as a mean-reverting stochastic process. A natural candidate is the Ornstein-Uhlenbeck (OU) process:

$$dS_t = \kappa(\mu - S_t)\,dt + \sigma\,dW_t,$$

where $\mu$ is the long-run mean, $\kappa$ the speed of mean reversion, and $\sigma$ the diffusion volatility.

This framework gives more structure than a purely empirical $z$-score rule. In particular, it allows us to estimate the half-life of mean reversion, which is useful for:

- evaluating whether a pair reverts fast enough to be tradable;
- calibrating entry and exit thresholds;
- setting holding-period expectations and stop-loss logic.

We would like to compare simple rolling-standardization signals with OU-based signals and assess whether the additional modeling improves robustness and economic significance.

### 3.3 Hurst exponent as a regime filter

The Kalman filter improves how we construct the spread and the OU process provides a structural model for its dynamics, but neither addresses a fundamental question: is the spread actually mean-reverting right now? A $z$-score may signal entry when the spread is far from its rolling mean, yet if the spread is in a trending regime, this deviation may widen rather than close.

The Hurst exponent $H$ captures the memory structure of a time series. For a spread $S_t$, it characterizes the scaling behavior of increments across time lags $\tau$:

$$\mathbb{E}\left[|S_{t+\tau} - S_t|^q\right] \propto \tau^{qH}.$$

The value of $H$ classifies the regime of the spread. When $H < 0.5$, the spread is anti-persistent: an upward move is more likely to be followed by a downward move, which is the mean-reverting behavior that pairs trading requires. When $H = 0.5$, increments are independent and the spread behaves as a random walk. When $H > 0.5$, the spread is persistent and trending, meaning that deviations from the mean tend to amplify rather than correct.

We propose to use the rolling Hurst exponent as a trade filter layered on top of the existing signal. Concretely, we compute $H_t$ over a rolling window of the spread and impose an additional entry condition: a trade is only opened when $|Z_t| > z^\star$ *and* $H_t < 0.5$. The Hurst filter does not generate signals; it decides entries in regimes where mean-reversion is unlikely, reducing exposure to trades that diverge rather than converge.

## 4 Application: from liquid to illiquid markets

Our implementation proceeds in two stages. We first validate the strategy on US-listed ETFs, where liquidity is deep and existing results provide a natural benchmark. We then apply the identical pipeline to Swiss equities listed on SIX.

The US stage serves as a proving ground. We select ETF pairs across asset classes using cointegration tests, and compare four strategy variants: static vs. Kalman-filtered hedge ratios, with and without the Hurst filter.

The Swiss stage provides a contrasting environment. The SMI contains a smaller pair universe, and transaction costs are slightly higher. To our knowledge, no dedicated study of pairs trading on Swiss equities exists in the literature.

The key hypothesis is that the Hurst filter should matter more in the Swiss market. In a liquid market, a false entry is cheap. In a thinner market, every trade that fails to revert incurs meaningful friction. We therefore expect the performance gap between filtered and unfiltered strategies to be larger on Swiss equities, suggesting that regime-aware filtering becomes increasingly important as liquidity declines.

## 5 Implementation pipeline

The project is organized as a modular pipeline:

1. **Data collection and cleaning:** retrieve price data, align timestamps, handle missing values, and prepare clean spread inputs;
2. **Pair selection:** screen candidate pairs using economic intuition, correlation filters, and cointegration tests;
3. **Model estimation:** estimate static OLS hedge ratios, dynamic Kalman-filter hedge ratios, and OU parameters;
4. **Signal generation:** compare rolling $z$-score rules with model-based entry/exit rules;
5. **Backtesting:** simulate execution with transaction costs, position sizing, and risk controls;
6. **Evaluation:** compare strategies using return, Sharpe ratio, drawdown, turnover, and robustness across subperiods.

For the first phase, we want a clean baseline pipeline from data sourcing to pair selection, signal generation, and backtesting. Once this benchmark is stable, we will incrementally test the Kalman-filter and OU extensions.

## 6 Conclusion

This project extends the classical pairs trading framework along two dimensions: spread construction, through a Kalman-filtered dynamic hedge ratio, and trade timing, through a Hurst exponent regime filter. By combining these extensions within a single modular pipeline, we aim to produce a strategy that adapts to changing market conditions and avoids entering trades in non-mean-reverting regimes. Testing on both US ETFs and Swiss equities will allow us to assess whether the benefits of regime-aware filtering scale with market illiquidity, and to provide, to our knowledge, the first dedicated pairs trading study on the Swiss market.